Color Superconductivity in a Strong Magnetic Field

Cristina Manuel
Instituto de Ciencias del Espacio (IEEC/CSIC)

E. Ferrer, V. de la Incera, C.M. PRL95 (2005) 152002;
E. Ferrer, V. de la Incera, C.M. hep-ph/0603233; C.M., hep-ph/0512054

SEWM 2006, BNL

Introduction

- The QCD phase diagram at low T and high μ is very rich - many different color superconducting phases, depending on the value of quark masses and chemical potentials (see Shovkovy's talk).

Introduction

- The QCD phase diagram at low T and high μ is very rich - many different color superconducting phases, depending on the value of quark masses and chemical potentials (see Shovkovy's talk).
- An external magnetic field also affects the phase diagram!

Introduction

- The QCD phase diagram at low T and high μ is very rich - many different color superconducting phases, depending on the value of quark masses and chemical potentials (see Shovkovy's talk).
- An external magnetic field also affects the phase diagram!

Orders of Magnitude

All compact stars support magnetic fields

B $\sim 10^{12} - 10^{14}$ G in the surface of pulsars

$B \sim 10^{15} - 10^{16}$ G in the surface of magnetars

There is an upper limit to the star magnetic field
(compare gravitational and magnetic energies)

$$B_{\max} \sim 1.4 \times 10^{18} \left(\frac{M}{M_\odot}\right) \left(\frac{10\,\mathrm{km}}{R}\right)^2 G$$

Color Flavor Locking Phase

For $m_q \approx 0$ ($m_s < 2\sqrt{\mu\Delta}$).

$$\langle q_L^{ia} q_L^{jb} \rangle = \Delta_A \, \epsilon^{ijk} \epsilon_{abk}$$

$a, b = 1, 2, 3$ flavor indices $\qquad$ $i.j = 1, 2, 3$ color indices

(Alford, Rajagopal, Wilczek, '98)

Local and global symmetries are spontaneously broken

$$SU(3)_C \times SU(3)_L \times SU(3)_R \times U(1)_B \to SU(3)_{C+L+R}$$

$\longrightarrow$ several similitudes with vacuum QCD

- There are Goldstone bosons associated to SSB of chiral symmetry $(\pi^0, \pi^\pm, K^0, \bar{K}^0, K^\pm, \eta)$
- One Goldstone boson for the breaking of $U(1)_B$

CFL matter in a external magnetic field

Is a CFL color superconductor also an electromagnetic superconductor ?

CFL matter in a external magnetic field

Is a CFL color superconductor also an electromagnetic superconductor ? yes and no ...

CFL matter in a external magnetic field

Is a CFL color superconductor also an electromagnetic superconductor ? yes and no ...

The condensates $\langle qq \rangle$ also break spontaneously $U(1)_{\rm e.m.}$.

But there is a combination of electromagnetism and a $U(1)$ subgroup of $SU(3)$ that remains unbroken

- Seven gluons and one combination of gluon and photon are massive.
- One combination of gluon and photon is massless

$$\widetilde{G}^8_\mu = \cos\theta_{\rm CFL}\, G^8_\mu + \sin\theta_{\rm CFL}\, A_\mu \ ,$$

$$\widetilde{A}_\mu = -\sin\theta_{\rm CFL}\, G^8_\mu + \cos\theta_{\rm CFL}\, A_\mu \, .$$

$$\cos\theta_{\rm CFL} = \frac{\sqrt{3}g}{\sqrt{3g^2 + 4e^2}}$$

CFL and the in-medium electromagnetism

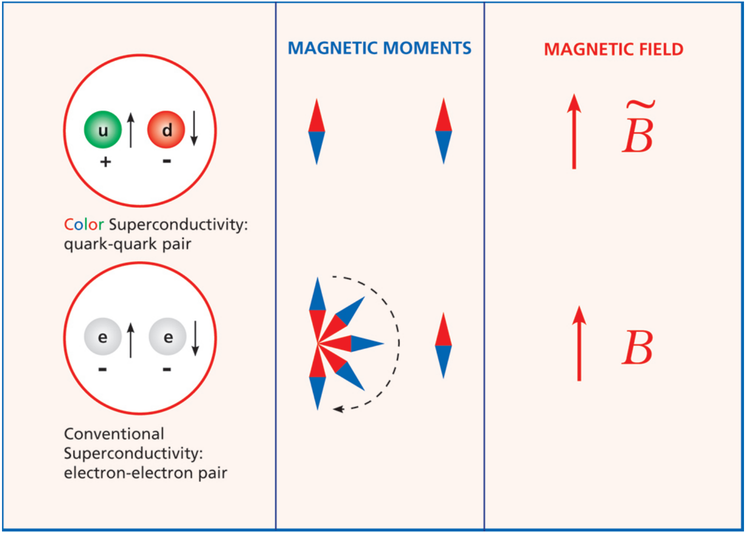
All quark, gluon and meson charges are integral

$$\tilde{e} = e\cos\theta_{CFL}$$

s_1	s_2	s_3	d_1	d_2	d_3	u_1	u_2	u_3
0	0	-	0	0	-	+	+	0

the "rotated" photon is massless; but the medium still modifies its propagation properties
$\tilde{v} = 1/\sqrt{\tilde{\epsilon}} < 1$ Litim and C.M. 01
the CFL medium acts as a transparent insulator, with interesting reflexion/refraction properties
C.M. and Rajagopal, 01

Influence of a Strong Magnetic Field in Superconductivity



Influence of B in fermion pairing

- In an electromagnetic superconductor
  a strong B field tends to break the condensate
- In the CFL color superconductors
  the penetrating field tends to stabilize the condensate.
- Magnetic catalysis of a chiral (fermion-antifermion) condensate at zero density , even at weak coupling
  Gusynin, Miransky and Shovkovy, 94,
  $\Rightarrow$ dimensional reduction of the pairing dynamics at the lowest Landau level

MCFL

Writing only the antisymmetric gaps

$$\langle q_L^{ia} q_L^{jb} \rangle = \Delta_A \, \epsilon^{ij3}\epsilon_{ab3} + \Delta_A^B \left(\epsilon^{ij1}\epsilon_{ab1} + \epsilon^{ij2}\epsilon_{ab2}\right)$$

Symmetry breaking pattern

$$SU(3)_C \times SU(2)_L \times SU(2)_R \times U^{(-)}(1)_A \times U(1)_B \times U(1)_{\mathrm{e.m.}} \\ \to SU(2)_{C+L+R} \times \tilde{U}(1)_{\mathrm{e.m}}$$

$\to$ several similitudes with vacuum QCD in an external B

Miransky and Shovkovy (2002)

- There are Goldstone bosons associated to SSB of $SU(2)$ chiral symmetry and $U^{(-)}(1)_A$ $(\pi^0, K^0, \bar{K}^0, \eta)$
- One Goldstone boson for the breaking of $U(1)_B$

Color Superconductivity

- Because of asymptotic freedom of QCD one can compute all the properties of the color superconductors for sufficiently high densities

Color Superconductivity

- Because of asymptotic freedom of QCD one can compute all the properties of the color superconductors for sufficiently high densities
- Use a Nambu-Jona-Lasinio (NJL) model, inspired by one-gluon exchange, to study more moderate densities (Λ UV cutoff)

$$\mathcal{L}_I = \frac{g^2}{\Lambda^2}\,\bar{\psi}\gamma^\mu\lambda^A\psi\bar{\psi}\gamma_\mu\lambda^A\psi$$

$$\mu \sim 400 - 500\,\mathrm{MeV}\ , \qquad \Delta \sim 10 - 50\,\mathrm{MeV}$$

Color Superconductivity

- Because of asymptotic freedom of QCD one can compute all the properties of the color superconductors for sufficiently high densities
- Use a Nambu-Jona-Lasinio (NJL) model, inspired by one-gluon exchange, to study more moderate densities (Λ UV cutoff)

$$\mathcal{L}_I = \frac{g^2}{\Lambda^2}\,\bar{\psi}\gamma^\mu\lambda^A\psi\bar{\psi}\gamma_\mu\lambda^A\psi$$

$$\mu \sim 400 - 500\,\mathrm{MeV}\ , \qquad \Delta \sim 10 - 50\,\mathrm{MeV}$$

- Interesting gluon dynamics at weak coupling and moderate fields see Ferrer and Incera 06

Some few technical details: Ritus method

$$\Pi_\mu^{(\pm)} = i\partial_\mu \pm \widetilde{e}\widetilde{A}_\mu$$

$$(\Pi^{(\pm)} \cdot \gamma) E_q^{(\pm)}(x) = E_q^{(\pm)}(x)(\gamma \cdot \overline{p}^{(\pm)})$$

$\overline{p}^{(\pm)} = (p_0, 0, \pm\sqrt{2|\widetilde{e}\widetilde{B}|k}, p_3)$ $\qquad$ k labels the Landau levels

$$E_q^{(\pm)}(x) = \sum_\sigma E_{q\sigma}^{(\pm)}(x)\Delta(\sigma)$$

$$\Delta(\sigma) = \mathrm{diag}(\delta_{\sigma 1}, \delta_{\sigma -1}, \delta_{\sigma 1}, \delta_{\sigma -1}), \qquad \sigma = \pm 1$$

$$E_{p\sigma}^{(\pm)}(x) = \mathcal{N}_{n_{(\pm)}} e^{-i(p_0 x^0 + p_2 x^2 + p_3 x^3)} D_{n_{(\pm)}}(\varrho_{(\pm)}) \ ,$$

$D_{n_{(\pm)}}(\varrho_{(\pm)})$: parabolic cylinder functions

$$\varrho_{(\pm)} = \sqrt{2|\widetilde{e}\widetilde{B}|}(x_1 \pm p_2/\widetilde{e}\widetilde{B}) \ ,$$

MCFL

We have solved the gap equations in an effective NJL model, inspired by one-gluon exchange, and for strong magnetic fields
$\tilde{e}\tilde{B} > \mu^2/2$
(then all the charged quarks are in the lowest Landau level)

$$\Delta_A^B \sim 2\mu \; \exp\left(- \frac{3\Lambda^2\pi^2}{g^2\left(\mu^2 + \widetilde{e}\widetilde{B}\right)}\right)$$

and $\Delta_A \ll \Delta_A^B$
to be compared with the CFL fermionic gap

$$\Delta_A^{\mathrm{CFL}} \sim 2\sqrt{\delta\mu} \; \exp\left(- \frac{3\Lambda^2\pi^2}{2g^2\mu^2}\right)$$

Magnetic catalysis of the diquark condensate

BCS behaviour of the gap

$$\Delta \propto \exp\left(-1/G^2\rho\right)$$

ρ : density of states close to the Fermi surface

a strong magnetic field increases the density of states of the charged quarks close to the Fermi surface!
from $\mu^2/2\pi^2 \Rightarrow \widetilde{e}\widetilde{B}/2\pi^2$.
orders of magnitude for the effect to be relevant
$\widetilde{e}\widetilde{B} \sim 10^{18}G$
for astrophysical applications one should look to more moderate fields - solving the gap equations for moderates fields requires a numerical analysis, with the inclusion of higher Landau levels

MCFL Low energy effective field theory

$$\Sigma = XY^\dagger = \exp\left(i\frac{\Phi}{f_{\pi,B}} + i\phi_0\right)\,, \qquad \Phi = \phi_A\sigma^A\,, \qquad A = 1,2,3$$

The external magnetic field introduces a strong anisotropy in the system

$$\mathcal{L} = \frac{f_{\pi,B}^2}{4}\left(\mathrm{Tr}\left(\partial_0\Sigma\partial_0\Sigma^\dagger\right) + \left(v_\perp^2 g_\perp^{ij} + v_\parallel^2 g_\parallel^{ij}\right)\mathrm{Tr}\left(\partial_i\Sigma\partial_j\Sigma^\dagger\right)\right)$$

The parameters of the low energy effective theory have to be computed!

Estimates

For which values of the magnetic field the superconductor is in CFL or MCFL phases?

Estimates

For which values of the magnetic field the superconductor is in CFL or MCFL phases?

Estimate based on the low energy effective field theory C.M.,05

CFL: 9 Goldstone bosons → MCFL: 5 Goldstone bosons

the magnetic field makes the charged GB $\pi^{\pm}, K^{\pm}$ **massive**

$$M^2_{\pi^\pm} = M^2_{K^\pm} \propto \frac{(\tilde{e}\widetilde{B}^{\rm ext})^2}{f_\pi^2}$$

when the mass of those are of order 2Δ they decouple

$$\tilde{e}\widetilde{B} \sim 2 f_\pi \Delta \ , \qquad \tilde{e}\widetilde{B} \sim 10^{16}\, G$$

Conclusions

- An applied external strong magnetic field leads to a new color superconducting phase. B has both qualitative and quantitative effects on color superconductivity!
  The low energy properties, including transport properties, interactions with neutrinos, cooling, etc, will differ.
- The dynamics of the magnetic field will also be so different.